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Stat 310, Part II, Optimization. Homework 4.

Problem 1: (computation;projected gradient for QP)

Implement Gradient Projection Algorithm 16.5 from the textbook (use a version of the truncated conjugated gradient approach that you implemented in Homework 3 to solve subproblem 16.74, instead of checking if the point is inside the trust region, check if it satisfies the bound constraints of 16.74).

Apply it to the following problem:

$$\min_x \frac{1}{2} x^T Q x - x^T f$$

$$l_i \leq x_i \leq u_i$$

where f=100*ones(n,1), l=0.5*ones(n,1), u=3.5*ones(n,1) and Q is the hessian of the cute problem from previous homework computed at ones(n,1). Start the algorithm from the point 2*ones(n,1). Record the number of matrix-vector multiplications for increasing values of n (start at about 10). For sanity, include a pseudocode of the overall algorithm.

Problem 2: (theory, quadratic programs, problem 16.22 from the textbook.)

Explain why, for bound constrained problems, the number of possible active sets is at most 3^n.

Problem 3: (theory, optimality conditions). Problem 12.19 from the textbook.

Consider the problem:

$$\min_{x \in \mathbb{R}^2} f(x) = -2x_1 + x_2 \text{ subject to } \begin{cases} (1-x_1)^3 - x_2 \geq 0 \\ x_2 + 0.25x_1^2 - 1 \geq 0 \end{cases}$$

The optimal solution is $x^* = (0,1)^T$ at which both constraints are active.

1. Do the LICQ conditions hold at this point?
2. Are the KKT conditions satisfied?
3. Write down the sets $\mathcal{F}(x^*)$ and $\mathcal{C}(x^*, \lambda^*)$
4. Are the second-order necessary conditions satisfied? Are the second-order sufficient conditions satisfied ?